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Sl.No.

M-793

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I Semester I B.Sc. Examination, March/April - 2021
(Semester Scheme) (CBCS) (2018-19 Batch and Onwards)
MATHEMATICS (Paper- I)
Algebra - I & Calculus - I

Time : 3 Hours **Max. Marks : 80**

Instructions:
1) **Answer all the Five questions.**
2) **First question carries 20 marks and remaining questions carry 15 marks.**

1. Answer any ten questions. Each question carries two marks.

a) Using elementary transformations show that

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix} \cong \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$ using elementary row operations.

c) If λ is an eigen value of the matrix A. Prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

d) If sum of the roots is $\frac{-5}{2}$ and product of the roots is 3. Find the quadratic equation.

e) If α, β, γ, are the roots of the equation $x^3 - 5x + 4 = 0$, find $\sum \alpha^2$.

f) Increase the root of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$ by 2.

g) Find the n^{th} derivative of $\log(2x+3)$.

h) Find the value of x for which. $x^3 + 8x^2 + 5x - 2$. is increasing.

i) Evaluate $\int_0^{\pi/2} \sin^5 x \, dx$.

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j) Find ϕ for the curve $r = a\,(1+\cos\theta)$

k) Find $\frac{ds}{dt}$ for the curve $x = a\,(t + \sin t),\ y = a\,(1 - \cos t)$.

l) Show that the radius of curvature at any point on the circle $x^2 + y^2 = a^2$ is a constant.

2. Answer any three questions. Each question carries five marks.

a) Find the rank of the matrix $\begin{bmatrix} 1 & -1 & -1 & 2 \\ 2 & 2 & 1 & -1 \\ 1 & -3 & 2 & 1 \\ 1 & -3 & 6 & -1 \end{bmatrix}$.

b) Find the values of λ and μ so that the system of equations.

$x + 2y + 3z = 4$

$x + 3y + 4z = 5$

$x + 3y + \lambda z = \mu$ have (1) no solution (2) a unique solution and 3) an infinite number of solutions.

c) Test for consistency and solve:

$3x - 2y - w = 2$

$2y + 2z + w = 1$

$x - 2y - 3z + 2w = 3$

$y + 2z + w = 1$

d) Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$.

e) Verify cayley - Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ and hence find A^{-1}.

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3. Answer any three questions. Each question carries five marks.

   a) Solve $x^4 - 2x^3 - 3x^2 + 4x - 1 = 0$, given that the product of two of its roots is 1.
   b) Test for multiple roots and solve $x^4 + 4x^3 + 3x^2 - 4x - 4 = 0$.
   c) Solve $6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0$.
   d) Solve $x^3 - 21x - 344 = 0$ by cardon's method.
   e) Solve $x^4 - 10x^2 - 20x - 16 = 0$ by Descarte's method.

4. Answer any three questions. Each question carries five marks.

   a) If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.
   b) Find the maximum and minimum values of $y = 2x^3 - 15x^2 + 36x + 10$.
   c) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ concave upwards or downwards also find its points of inflexion.
   d) Evaluate $\int_0^{\pi} x\sin^6 x\cos^4 x\, dx$.
   e) Prove that $\int_0^{\pi/2} \sin^n x\, dx = \frac{(n-1)(n-3)....}{n(n-2)....}K$ where $K = 1$ or $\frac{\pi}{2}$ according as n is odd or even.

. Answer any three questions. Each question carries five marks.

   a) Find the angle between the curves $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$.
   b) Show that the pedal equation of the curve $x^2 + y^2 = 2ax$ is $r^2 = 2ap$.
   c) Prove with usual notation that $\rho = \frac{\left(1+y_1^2\right)^{3/2}}{y_2}$
   d) Find the radius of curvature for the curve $x = a\cos^3 t$, $y = a\sin^3 t$ at any point on it.
   e) Find the evolute of the parabola $y^2 = 4ax$.

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