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II Semester I Year B.Sc. Examination, July/August - 2023

(Semester Scheme) (NEP)

MATHEMATICS

Algebra - II and Calculus - II (Paper-II) (DSC)

Time : 2½ Hours **Max. Marks : 60**

Instructions : *1) Answer all the questions.*
2) All the questions carry equal marks.

1. Answer any three of the following : **[3 × 5 = 15]**

   a) Find the number of positive divisors and sum of positive divisors of the composite number 960.

   b) Find the G.C.D. of 222 and 469. Also express G.C.D. as a linear combination of the given numbers.

   c) Solve the simultaneous congruences $x \equiv 2 \pmod 3$ and $x \equiv 3 \pmod 5$.

   d) i) Find the least non-negative remainder when 3^{200} is divided by 5

      ii) Find $\phi(1155)$

   e) State and prove Euler's theorem.

2. Answer any three of the following : **[3 × 5 = 15]**

   a) Show that $f(x) = \begin{cases} x^2 + 2, \text{ when } x > 1 \\ 2x + 1, \text{ when } x = 1 \\ 3 \text{ , when } x < 1 \end{cases}$ is continuous at $x = 1$.

   b) Examine the differentiability of the function

$$f(x) = \begin{cases} x^2, \text{ if } x \le 3 \\ 6x - 9, \text{if } x > 3 \text{ at } x = 3 \end{cases}$$

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c) State and prove Lagrange's mean value theorem.

d) Expand log (1 + sin x) up to the term containing x^4 using Maclaurin's expansion.

e) Evaluate $\lim_{x \to 0}\left[\cot x - \frac{1}{x}\right]$

3. Answer any three of the following : [3 × 5 = 15]

a) If $u(x,y) = \sin^{-1}\left[\frac{x^3 - y^3}{x + y}\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

b) If $z = x^2 + y^{-2}, x = e^t \cos t, y = 1 + \log t$, find $\frac{dz}{dt}$.

c) Find the second Taylor polynomial of $f(x,y) = \log(1 + x + y)$ at $x = y = 0$.

d) If $x = r\cos\theta, y = r\sin\theta$, find J and J' and also verify JJ' = 1.

e) Test for maximum and minimum of $z = f(x,y) = x^3 + y^3 - 3xy$.

4. Answer any three of the following : [3 × 5 = 15]

a) Evaluate $\int_C (x^2 - y)dx + (y^2 + x)dy$ where $C: x = t, y = t^2 + 1$ and $0 \le t \le 1$.

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dxdy$ by changing the order of integration.

c) Find the area of the surface $z = \sqrt{x^2 + y^2}, \frac{1}{16} < x^2 + y^2 < \frac{1}{4}$

d) Evaluate $\iiint_D xy^2z^3 dxdydz$ where D is given by $0 < x < 10, 0 < y < 1, 0 < z < 1$.

e) Find the volume of tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

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