93708 | **MP-465**

Sl. No. | **Total No. of Pages : 4**

VI Semester III B.Sc. Examination, July/August - 2023
(Semester Scheme) (CBCS)
MATHEMATICS
DSE : Algebra - IV and Complex Analysis - I

Time : 3 Hours | **Max. Marks : 80**

Instructions : *1) Answer all the five questions.*

2) First question carries 20 marks remaining questions carry 15 marks.

1. Answer any Ten questions. Each question carries two marks :

   a) In any vector space V(F), Prove that $C(\alpha - \beta) = C\alpha - C\beta$ for all $C \in F$, $\alpha, \beta \in V$.

   b) Show that the vectors (a_1, a_2) and (b_1, b_2) in $V_2(F)$ are linearly dependent if any only if $a_1b_2 - a_2b_1 = 0$.

   c) Determine whether $S = \{(x, y, z) \mid x = 0 \text{ or } y = 0\}$ is a subspace of $V_3(R)$ or not.

   d) Is the transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x+2, y+3)$ linear? Justify.

   e) Find $T^2(x, y)$ of the linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (-x, y)$.

   f) Find the matrix of linear transformation $T(x, y, z) = (-x + y + z, x + y - z)$ w.r.t. standard bases.

   g) Find the modulus-argument of $\sqrt{3} - i$.

   h) Evaluate $\lim\limits_{z \to e^{i\pi/4}} \dfrac{z^2}{z^4 + z + 1}$

P.T.O.

i) Show that the function $f(z) = e^z$ is analytic, where $z = x + iy$.

j) Define cross ratio of four points.

k) Find the Jacobian of the transformation $w = (x + y) + i(x - y)$.

l) Find the fixed points of the transformation $w = \frac{3z - 4}{z}$.

2. Answer any three questions. Each question carries five marks :

a) Show that the set of ordered pair of real numbers is a vector space over the field of real numbers.

b) If S and T are any two subspaces of a finite dimensional vector space then prove that $d[S] + d[T] = d[S+T] + d[S \cap T]$.

c) Construct an addition table for $V_2(z_2)$ and list all its subspaces.

d) Find the basis and dimension of the subspace spanned by the vectors $(2, 4, 2)$, $(1, -1, 0)$ and $(0, 3, 1)$.

e) If n vectors spans a vector space V containing r linearly independent vectors then prove that $n \geq r$.

3. Answer any three questions. Each question carries five marks :

a) Find a linear transformation

$T : V_2(R) \rightarrow V_2(R)$ such that $T(2, 1) = (3, 4)$ and $T(-3, 4) = (0, 5)$.

b) Find the matrix of a linear transformation $T : V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (-x + 2y, y, -3x + 3y)$ relative to the bases $B_1 = \{(1, 1), (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.

c) Find the range, kernel, rank and nullity of the linear transformation $T : V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (x, x+y, y)$.

d) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$ using linear transformation.

e) Find the eigen values and eigen vectors of the linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (3x + 3y, x + 5y)$.

4. Answer any three questions. Each question carries five marks :

a) Derive the equation of a straight line in complex form and hence express $3x + 4y = 5$ in complex form.

b) Find the equation of a circle passing through the points $1 + i$, $1 - i$ and 2.

c) Find the derivative of $f(z) = \dfrac{2z+1}{2z-1}$ at $z = a$ using definition of derivative.

d) Find the analytic function whose real part is $x^2 - y^2 - c$.

e) Define harmonic function. Prove that the real and imaginary parts of an analytic function are harmonic.

5. Answer any three questions. Each question carries five marks :

a) Prove that a bilinear transformation establishes a one-one correspondence from the extended z-plane to extended w-plane.

b) Prove that $w = \dfrac{i(z-i)}{z+i}$ maps the upper half of the z-plane into the interior of the unit circle in the w-plane.

c) Find a bilinear transformation which maps the points $(-1, -i, -1)$ onto $(1, 0, i)$.

d) Discuss the transformation $w = \cos z$.

e) Express the bilinear transformation $w = \frac{3z+2}{4z+3}$ as a resultant of translation, rotation, magnification and inversion.

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