93709 MP-466

Sl.No.

Total No. of Pages : 2

VI Semester III Year B.Sc. Examination, July/August - 2023

(Semester Scheme) (CBCS)

MATHEMATICS

Complex Analysis - II and Improper Integrals (SEC)

Time : 2 Hours **Max. Marks : 40**

Instructions : *1) Answer all the questions.*

2) First question carries 10 marks and remaining questions carry 15 marks.

1. Answer any five questions. Each question carries two marks.

   a) Evaluate $\int_C (x+2y)dx+(3y-x)dy$ along the curve $y=x^2$ from (0, 0) to (1, 1).

   b) Evaluate $\int_C (\bar{z})^2 dz$ where C is the circle $|z|=1$.

   c) Evaluate $\int_C \frac{\sin 3z}{z-\pi/2} dz$ where C is the circle $|z|=5$.

   d) State Cauchy's inequality.

   e) Prove that $\int_0^1 \left[\log\left(1/y\right)\right]^{n-1} dy=\Gamma(n)$.

   f) Evaluate $\int_0^\infty x^3 e^{-x} dx$.

   g) Evaluate $\beta(3, 5)$.

   h) Prove that $\Gamma(1/4)\Gamma(\frac{3}{4})=\sqrt{2}\pi$ using duplication formula.

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2. Answer any three questions. Each question carries five marks.

a) Evaluate $\int_{(0,3)}^{(2,4)}\left[\left(2y+x^2\right)dx+\left(3x-y\right)dy\right]$ along the curve $x = 2t$ and $y = t^2 + 3$.

b) If $f(z)$ is analytic over a simply connected region R and $z = a, z = b$ are two points in R, then show that $\int_a^b f(z)dz$ is always independent of the path joining the points a and b.

c) Evaluate $\oint_C \frac{z^2-4}{z\left(z^2+9\right)}dz$ where C is the circle $|z| = 1$.

d) Evaluate $\oint \frac{(z-3)}{(z+1)^2(z+2)}dz$ where C is the circle $|z| = 1$.

e) State and prove Liouville's theorem.

3. Answer any three questions. Each question carries five marks.

a) Evaluate $\int_0^1 \frac{dx}{\sqrt{x\log\left(\frac{1}{x}\right)}}$.

b) Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

c) Prove that $n\beta(m+1,n)=m\beta(m,n+1)$.

d) Prove that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$.

e) Show that $\int_0^{\pi/2}\sqrt{\cos\theta}\,d\theta \int_0^{\pi/2}\frac{1}{\sqrt{\cos\theta}}d\theta=\pi$

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