

Sl.No.

Total No. of Pages : 3

VI Semester B.Sc. Examination, April/May - 2018

(Semester Scheme)

MATHEMATICS (Paper - VII)

(2015-16 Batch and onwards)

Algebra IV and Calculus III

Time : 3 Hours

Max. Marks : 80

Instructions : Answer all the sections.

SECTION - A

I. Answer any eight questions. Each question carries two marks.

- Prove that the vectors  $(1,2,3)$ ,  $(1,1,1)$  and  $(0,1,0)$  are linearly independent.
- Express  $(1,7,-4)$  as a linear combination of  $(1,-3,2)$  and  $(2,-1,1)$  in  $V_3(R)$ .
- Determine whether  $S = \{(x, y, z) / x, y, z \in R, x = y = z\}$  is a subspace of  $V_3(R)$
- Discuss the linearity of  $T: R \rightarrow R^2$  defined by  $T(x) = (x + 1, x + 5)$ .
- Find the matrix of the linear transformation  $T(x, y, z) = (-x + y + z, x - y + z)$  where  $T: V_3(R) \rightarrow V_2(R)$ .
- Find  $T^2(x, y)$  of  $T: V_2(R) \rightarrow V_2(R)$  defined by  $T(x, y) = (x, x - y)$
- Prove that  $\Gamma(n + 1) = n\Gamma(n)$
- Prove that  $\beta(m, n) = \beta(m + 1, n) + \beta(m, n + 1)$
- Find  $\int_0^1 x^2(1 - x)^3 dx$  using Beta function.
- If  $\phi = \log(x^2 + y^2 + z^2)$  then find  $\Delta\phi$  at  $(1, 1, 1)$ .
- Show that the vector  $(6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$  is irrotational
- If  $\vec{F} = x^2y\hat{i} + 2xyz\hat{j} + y^2z\hat{k}$  find  $\text{div}\vec{F}$

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**SECTION - B**

II. Answer any eight questions. Each question carries four marks..

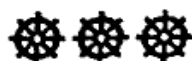
- Show that the set  $Q(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in Q\}$  is a vector space over  $Q$  under the operations usual addition and scalar multiplication
- Show that the set  $\{(1,1,0), (1,0,1), (0,1,1)\}$  forms a basis of the vector space  $V_3(R)$ .
- Show that any two bases of a finite dimensional vector space  $V$  have the same finite number of vectors.
- Construct the addition table for  $V_2(Z_2)$  and list all its subspaces.
- If 'n' vectors spans a vector space  $V$  containing 'r' linearly independent vectors in  $V$ , then prove that  $n \geq r$
- Show that  $T: R^3 \rightarrow R^2$  defined by  $T(x,y,z) = (x+z, x+y+z)$  is a linear transformation. <https://www.uomonline.com>
- Find a linear transformation  $T: V_2(R) \rightarrow V_2(R)$  such that  $T(2,1) = (3,4)$ ,  $T(-3,4) = (0,5)$ .
- Find the range, rank, kernel, null space of the linear transformation  $T: V_2(R) \rightarrow V_3(R)$  defined by  $T(x,y) = (x-y, y, x+y)$
- Prove that every vector space  $V$  over  $F$  of dimension  $n$  is isomorphic to  $V_n(R)$ .
- Show that the linear map  $T: V_3(R) \rightarrow V_3(R)$  defined by  $T(e_1) = e_1 + e_2$ ,  $T(e_2) = e_2 + e_3$ ,  $T(e_3) = e_1 + e_2 + e_3$  is non singular and find its inverse.

**SECTION - C**

III. Answer any eight questions. Each question carries four marks.

- Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

- b) Evaluate  $\int_0^1 x^5(1-x^3)^{10} dx$
- c) Show that  $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} d\theta \times \int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta = \pi$
- d) Show that  $\int_0^{\infty} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx = 2\beta(m, n)$
- e) Prove that  $\frac{\Gamma\left(n + \frac{1}{2}\right)}{\Gamma(n+1)} = \frac{1.3.5...(2n-1)}{2.4.6.....2n} \sqrt{\pi}$
- f) Find the directional derivative of the function  $\phi = x^2y + y^2z - xyz$  at the point  $(2, -4, 6)$  in the direction of the vector  $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ .
- g) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  show that  $\text{div} [f(r)\vec{r}] = rf'(r) + 3f(r)$
- h) If  $\vec{f} = xyz\hat{j}$  and  $\vec{g} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$  find
- $\text{div} (\vec{f} \times \vec{g})$
  - $\Delta \text{div} (\vec{f} \times \vec{g})$
- i) If  $\phi$  is a scalar function and  $\vec{A}$  is a vector function then prove that  $\text{div} (\phi \vec{A}) = \text{grad } \phi \cdot \vec{A} + \phi (\text{div } \vec{A})$
- j) Verify Green's theorem for the function  $P = xy + y^2$ ,  $Q = x^2$  over the closed curve C of the region bounded by  $y = x^2$  and  $y = x$ .



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