20763 **M-5591**

Sl.No. **Total No. of Pages : 3**

VI Semester B.Sc. Examination, April/May - 2019
(Semester Scheme) (2015 - 16 Batch and Onwards)
MATHEMATICS (Paper - VII)
Algebra IV & Calculus III

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer all the sections.

SECTION - A

I. Answer any eight questions. Each question carries two marks.

a) In a vector space V over the field F, if α, β ∈ V and $a \neq 0 \in F$. Then show that $a\alpha = a\beta \Rightarrow \alpha = \beta$.

b) Prove that the set {(1, 2, 1), (–1, 1, 0), (5, –1, 2)} of vectors is a basis of $V_3(R)$.

c) Define direct sum of subspaces of a vector space.

d) If $T: V \rightarrow V$ is a linear transformation, then show that T(O) = O'.

e) Find the Eigen values of $T: R^2 \rightarrow R^2$ defined by T(1, 0) = (1, 2) and T(0, 1) = (4, 3).

f) Find $T^2(x, y, z)$ of the transformation $T: V_3(R) \rightarrow V_3(R)$ defined by $T(x, y, z) = (z, -y, x)$.

g) Prove that $\Gamma(n+1) = n\Gamma n$.

h) Evaluate $\beta\left(\frac{1}{2}, \frac{3}{4}\right)$.

i) Evaluate $\int_0^\infty x^4 e^{-x}\, dx$ using Gamma function.

j) Find the divergence of the vector field $\overline{f} = x^2 i + 3yj + z^3 k$.

P.T.O.

k) Show that $(x+4y)i+(2y-z)j+(x+y-3z)k$ is solenoidal.

l) Prove that $curl(grad\,\phi)=0$.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

a) Show that the set of all 2 × 2 matrices over the field of real numbers is a vector space.

b) If S and T are any two subspaces of a vector space V over the field F, then show that S + T is also a subspace of V over F.

c) In $V_3(z_3)$ how many vectors are spanned by (1, 2, 2) and (2, 1, 1).

d) Find a basis and dimension of the subspace of $V_3(R)$ spanned by the vectors (1, 1, 1), (1, 2, 3) and (–1, 0, 1).

e) Prove that $\dim\left(\frac{V}{W}\right)=\dim(V)-\dim(W)$ where W is a subspace of finite dimensional vector space V over the field F.

f) Show that $T:R^3\to R^2$ defined by $T(x,y,z)=(x+z,x+y+z)$ is a linear transformation.

g) Find the matrix of the linear transformation T(1, 1) = (0, 1) and T(–1, 1) = (3, 2).

h) Find a linear transformation $T:R^3\to R^2$ such that T(1, 0, 0) = (–1, 0) T(0, 1, 0) = (1, 1), T(0, 0, 1) = (0, –1).

i) Show that the sum of two linear transformations is also linear.

j) Find the range, kernal, rank and nullity of the linear transformation whose matrix is $\begin{bmatrix}1 & 3 & 2\\ -1 & 7 & 2\\ 1 & 0 & 1\end{bmatrix}$

SECTION - C

III. Answer any eight questions. Each question carries four marks.

a) For $m>0, n>0$ show that $\int_0^\infty \frac{x^{m-1}-x^{n-1}}{(1+x)^{m+n}}dx=0$.

b) Show that $\int_0^{\pi/2} \sin^m\theta \cos^n\theta d\theta = \frac{1}{2}\beta\left[\frac{m+1}{2},\frac{n+1}{2}\right]$.

c) Prove that $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$.

d) Evaluate $\int_0^1 x^3(1-x^2)^{5/2}dx$.

e) Prove that $\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)}=\frac{1.3.5......(2n-1)}{2.4.6......2n}\sqrt{\pi}$.

f) Show that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$ where $r^2=x^2+y^2+z^2$.

g) Find the directional derivative of the function $\phi=4xz^3-3x^2y^2z$ at $(2,-1,2)$ along $2\hat{i}-3\hat{j}+6\hat{k}$.

h) If $\bar{f}=x^2y\hat{i}+y^2z\hat{j}+z^2x\hat{k}$ find $curl(curl\,\bar{f})$ at $(1, 2, 1)$.

i) If $\bar{f}$ and $\bar{g}$ are two differentiable vector functions, then prove that $div(\bar{f}\times\bar{g})=\bar{g}.curl\,\bar{f}-\bar{f}.curl\,\bar{g}$.

j) Use Green's theorem, Evaluate $\int_C (xy-x^2)dx+x^2y\,dy$ along the closed curve C formed by $y=0$, $x=1$, $y=x$.

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