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M-22

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VI Semester B.Sc. Examination, April/May - 2018

(Semester : Scheme )

MATHEMATICS (Paper - VIII)

Complex Analysis and Numerical Analysis

(2015-16 Batch and Onwards)

Time : 3 Hours

Max. Marks : 80

Instruction: Answer all the sections.

SECTION - A

I. Answer any eight questions . Each question carries two marks.

- Find the equation of the line joining the points  $2+3i$  and  $1+2i$ .
- If  $f(z) = \sin z$ , find  $f'(z)$  at  $z = i$  using the definition of derivative.
- Prove that  $f(z) = z^2$  is analytic.
- Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the curve  $y = x$
- Evaluate  $\int_C \frac{1}{z-2} dz$  where  $C$  is the circle  $|z|=4$
- State fundamental theorem of Algebra.
- Find an interval in which a real root of  $x^3 - x - 4 = 0$  lies.
- Use Newton - Raphson method to find  $\sqrt{17}$  correct to three decimal Places.
- Solve  $\frac{dy}{dx} = y - x$ , by Picard's method upto two approximations given  $y(0) = 1$ .
- Prove that  $(1 + \Delta)(1 - \nabla) = 1$
- Construct the forward difference table for  $f(x) = x^3 + 1$  for  $x = 0 (1) 5$
- State simpson's  $\frac{1}{3}$ <sup>rd</sup> rule for 'n' intervals.

P.T.O.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

- a) Define continuity of  $f(z)$  at  $z = z_0$  and show that  $f(z) = \frac{z^2}{z^4 + z^2 + 1}$  is continuous at  $z = e^{i\pi/2}$ .
- b) Find whether the points  $(2,1)$ ,  $(3,5)$ ,  $(-2,0)$  and  $(1,-1)$  are concyclic or not.
- c) State and prove Cauchy-Riemann equations in polar form.
- d) Find the analytic function  $f(z)$  whose real part is  $e^x \cos y$  and find its imaginary part.
- e) Prove that the function  $u = x^3 - 3xy^2$  is harmonic and find its harmonic conjugate.
- f) State and prove Cauchy's integral formula.
- g) Evaluate  $\int_C (\bar{z})^2 dz$  around the circle  $|z-1|=1$ .
- h) Show that  $\int_C \frac{z^2-4}{z(z^2+9)} dz = \frac{-8\pi i}{9}$  where 'C' is the circle  $|z|=1$ .
- i) Evaluate  $\int_C \frac{z \cos z}{(z-\pi/2)^2} dz$  where C is the circle  $|z-i|=3$ .
- j) State and prove Cauchy's inequality.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

- a) Find a real root of the equation  $x^3 - 2x - 5 = 0$  by Bisection method correct to three decimal places.
- b) Find a real root of the equation  $\cos x - 3x + 1 = 0$  correct to three decimal places by the method of false position.

- c) Use modified Euler's method to solve  $\frac{dy}{dx} = x - y^2$ , given that  $y(0) = 1$  for  $x = 0.2$  with  $h = 0.1$

- d) Apply Runge - Kutta fourth order method to solve

$$\frac{dy}{dx} = 2x - y \text{ with } y(0) = 1, \text{ for } x = 0(0.5)1$$

- e) Estimate the population for the year 1995 from the given table.

Year	1960	1970	1980	1990	2000
Population in crores	46	66	81	93	101

- f) Use Newton - Gregory formula to find a polynomial in  $x$  for the data.

$x$	0	1	2	3
$f(x)$	2	3	12	35

- g) Using Lagrange's interpolation formula find  $f(5)$  given that  $f(1) = 2, f(2) = 4, f(3) = 8$  and  $f(7) = 128$

- h) Derive general quadrature formula.

- i) Evaluate  $\int_0^1 \frac{x}{1+x^4} dx$  with  $n = 4$  using trapezoidal rule, hence find an approximate value of  $\pi$ .

- j) Evaluate  $\int_0^3 (x^4 + x) dx$  with  $n = 6$  by using weddle's rule.

