20764 **M-3192**

Sl. No. Total No. of Pages : 3

VI Semester B.Sc. Examination, September - 2020
(Semester Scheme) (2015-16 Batch and onwards)
MATHEMATICS (Paper - VIII)
Complex Analysis and Numerical Analysis

Time : 3 Hours **Max. Marks : 80**

Instruction : *Answer all the Sections.*

SECTION - A

I. Answer any eight questions. Each question carries two marks.

a) Find the equation of the Straight line passing through the points $z_1 = 2 + i$ and $z_2 = 3 - 2i$.

b) Find the region of the complex plane satisfying $|z+1-i| \leq 3$.

c) Prove that $f(z) = \cos z$ is analytic.

d) Find $\int_c (z)^2 dz$ where c is the circle $|z|=1$.

e) Evaluate $\int_c \frac{e^z}{z^2} dz$ where C is a circle $|z|=1$.

f) State fundamental theorem of Algebra.

g) Construct forward difference table for $f(x) = x^2 + 3x + 1$ for the values $x = 0, 1, 2, 3, 4$.

h) Evaluate $\Delta^9 (1-3x^2)(1-5x^3)(1-7x^4)$.

i) Show that $E = 1 + \Delta$.

j) Use Euler-Cauchy method to solve $\frac{dy}{dx} = x + y$, given $y(0) = 1$ for $x = 0(0.1)0.2$.

P.T.O.

k) State Lagarange's interpolation formula for unequal intervals.

l) Evaluate $\int_0^3 \frac{dx}{1+x}$ with $n=3$ using Trapezoidal rule.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

a) Show that $\arg\left(\frac{z-1}{z+1}\right)=\pi/3$ represents equation of a circle. Find its centre and radius.

b) Evaluate $\lim_{z\to 1+i}\left\{\frac{z^2-z+1-i}{z^2-2z+2}\right\}$.

c) Find the derivative of $f(z)=\frac{2z-i}{z+2i}$ at $z = -i$ using the definition of derivative.

d) Derive Cauchy-Riemann equations in polar form.

e) Find the analytic function whose real part is $x^2 - y^2 - y$ hence find the imaginary part.

f) Evaluate $\int_c z\,dz$, where c is the square whose sides are $x=\pm1$, $y=\pm1$, described in the positive direction.

g) State and prove Cauchy's integral theorem.

h) Evaluate $\int_c \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$, where C is the circle $|z|=1$.

i) Show that $\int_c \frac{z^2-4}{z(z^2+9)}dz = \frac{-8\pi i}{9}$ where C is the circle $|z|=1$.

j) State and prove Cauchy's inequality.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

a) Find a real root of $x^3 - 2x - 5 = 0$ by bisection method correct to three decimal places.

b) Find a real root of the equation $x^3 - x - 1 = 0$ correct to three decimal places by Newton Raphson method.

c) Solve $\frac{dy}{dx} = x^2 + y^2$, $y = 0$ when $x = 0$ by Picard's method upto third approximation at $x = 0.2$.

d) Using Runge-Kutta fourth order method solve $\frac{dy}{dx} = 1 + xy, y(0) = 2$ for $x = 0.2, h = 0.1$.

e) Find the 10^{th} term of the series 3, 14, 39, 84, 155, 258 ....

f) Find y when $x = 0.33$ from the table by using Newton's Gregory formula.

x	0.30	0.40	0.50	0.60
y	0.6179	0.6554	0.6915	0.7257

g) Prove that $\left(\frac{\Delta^2}{E}\right)e^x \frac{Ee^x}{\Delta^2 e^x} = e^x$ where the interval of difference being unity.

h) Derive the Simpson's $1/3^{rd}$ Rule using General Quadrature formula for n intervals.

i) Evaluate $\int_0^6 \frac{dx}{x^2 + 4}$ by Weddle's rule with $n = 6$.

j) Use Simpson's $3/8^{th}$ rule to obtain an approximate value of $\int_{0.2}^{1.4} e^{2x}\, dx$ with $n = 6$.

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