

VI Semester B.Sc. Examination, April/May - 2019

(Semester Scheme)

MATHEMATICS (Paper - VIII) (2015-16 Batch and Onwards)

Complex Analysis and Numerical Analysis

Time : 3 Hours

Max. Marks : 80

Instruction : Answer all the sections.

SECTION - A

I. Answer any eight questions. Each question carries two marks.

- a) Evaluate $\lim_{z \rightarrow 2} \frac{z^3 - 2z^2 + 2z - 4}{z^2 - 3z + 2}$.
- b) Show that an analytic function with constant real part is constant.
- c) Prove that $u = \log \sqrt{x^2 + y^2}$ is harmonic.
- d) Evaluate $\int_C \operatorname{Re}(z) dZ$, where C is the line joining $Z = 0$ and $Z = 1+i$.
- e) Evaluate $\int_C \frac{dZ}{Z+2}$, around the circle $|Z| = 2$.
- f) State Cauchy's inequality.
- g) Explain briefly the Bisection method of finding a real root of the equation $f(x) = 0$.
- h) By using Newton-Raphson method find $\sqrt[3]{10}$ correct to three decimal places.
- i) Solve $\frac{dy}{dx} = x + y$, given $y(0) = 1$ using Picard's method up to 3rd approximation.
- j) Construct the forward difference table from.
- | | | | | | |
|---|---|---|---|----|----|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 3 | 7 | 13 | 21 |
- k) Evaluate $\Delta^{10} (1+3x)(1-9x^2)(1-6x^3)(1+5x^4)$
- l) State Simpson's $\frac{3}{8}$ th rule for n intervals.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

- Find the equation of the circle passing through the points 1, i, and $1+i$.
- Find the derivative of $f(Z) = \frac{Z-1}{Z+1}$ at $2-i$ using definition of derivative.
- Show that the real and imaginary parts of an analytic function are harmonic.
- If $f(z) = u + iv$ is analytic and $u-v = e^x [\cos y - \sin y]$. Find $f(z)$ in terms of z .
- If u and v are harmonic functions, show that $(u_x - v_y) + i(u_y + v_x)$ is analytic.
- Evaluate $\int_{(0,0)}^{(2,4)} (2y - x^2) dx + (3x - y) dy$ along the curve $x = 2t$, $y = t^2 + 3$.
- State and prove Cauchy's integral theorem.
- Evaluate $\int_C \frac{\cos(2\pi Z)}{(2Z-1)(Z-3)} dz$, where C is the circle $|z| = 1/2$.
- Evaluate $\int_C \frac{e^{-2z}}{(z+4)^3} dZ$, where c is the circle $|Z| = 4$.
- State and prove Liouville's theorem.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

- Find a real root of the equation $x^3 - 5x + 3 = 0$ correct to three decimal places using Bisection method. <https://www.uomonline.com>
- Find a real root of the equation $x^3 - x - 10 = 0$ correct to three decimal places by Regula-falsi method.
- Use Euler's modified method to compute $y(0.4)$ for $x = 0(0.2)0.4$ given that $\frac{dy}{dx} = 1 - 2xy$, $y(0) = 0$.

d) Solve $\frac{dy}{dx} = y - x$, given $y(0) = 0$, at $x = 0.4$ with $h = 0.2$ by Runge-Kutta fourth order method.

e) Estimate $f(2.5)$ from the given table by using forward interpolation formula.

x	1	2	3	4	5	6
f(x)	1	8	27	64	125	216

f) Express $\frac{3x^2 + x + 1}{(x-1)(x-2)(x-3)}$ as a sum of partial fractions by using Lagrange's interpolation formula.

g) Using Newton - Gregory formula. Find a polynomial $f(x)$ from the following table.

x	0	1	2	3
f(x)	1	2	4	7

h) Derive Simpson's $\frac{1}{3}$ rd rule using general quadrature formula for n intervals.

i) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Trapezoidal rule with $n = 5$, Hence find an approximate value of $\log 2$.

j) Find the value of $\int_0^{0.6} \frac{dx}{1+x^2}$ by using Weddle's rule with seven ordinates.

