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Sl.No.

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VI Semester B.Sc. Examination, May/June - 2016
(Semester Scheme)
MATHEMATICS (Paper - IX)
Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions :
1) *Section A is compulsory.*
2) *Answer any five full questions from sections B and C choosing atleast two from each section.*
3) *All questions in sections B and C carry equal marks.*

SECTION - A

Q1) Answer any ten questions. Each question carry two marks.

a) In a vector space V over the field F. Show that $c(\alpha - \beta) = c\alpha - c\beta \, \forall c \in F$ and $\alpha, \beta \in V$.

b) Express (3, 5, 2) as a linear combination of the vectors (1, 1, 0), (2, 3, 0), (0, 0, 1) in $V_3(R)$.

c) Show that the subset $W = \{(x, 0, y) / x, y \in R\}$ is a subspace of $V_3(R)$.

d) If the vectors (1, –2, 2), (3, 0, 4) (–2, a, –4) are linearly independent in $V_3(R)$ then find the value of a.

e) Show that the set of vectors {(1, 0), (0, 1)} form a basis of $V_2(R)$.

f) In any Euclidean vector space. Prove that $|c\xi| = |c| \, |\xi|$ for any scalar c and any vector ξ.

g) In a Euclidean vector space prove that $|\xi + \eta|^2 + |\xi - \eta|^2 = 2[|\xi|^2 + |\eta|^2]$

h) Find the orthogonal projection of (2, 4, 3) on the subspace spanned by the vector (1, 0, –1).

i) If W is the subspace of a vector space V and $W^{\perp}$ is the orthogonal complement of W then show $W \cap W^{\perp} = \{0\}$.

j) Define a linear transformation from a vector space to another vector space.

P.T.O.

k) Find $T^2(x, y, z)$ of the transformation $T : V_3(R) \to V_3(R)$ defined
$T(x, y, z) = (-z, y, -x)$

l) Find the matrix of a linear transformation $T : R^2 \to R^2$ given
$T(x, y) = (y, -x)$ relative to the standard basis.

m) Find the orthogonal complement of $S = \{(x, y, 0) / x, y \in R\}$.

n) Find the angle between the vectors (3, 2, −1, 4) and (0, 2, −3, 1).

o) State Rank - Nullity theorem of a linear transformation.

SECTION - B

Q2) a) Show that the set of complex numbers is a vector space over the field of real numbers.

b) Show that the set of all linear combinations of any given set of vectors in a vector space V will be a subspace of V.

c) In $V_3(z_3)$ how many vectors are spanned by (2, 1, 1) and (1, 2, 2).

Q3) a) Prove that any subset of a linearly independent set is linearly independent.

b) Find the basis and dimension of the subspace spanned by the vectors (2, 4, 2), (1, −1, 0) and (0, 3, 1)

c) Prove that any two bases of a finite dimensional vector space have same number of elements.

Q4) a) Prove that V is the direct sum of subspaces S and T if and only if V = S + T and $S \cap T = \{0\}$.

b) Show that any 'n' dimensional vector space over a field F is isomorphic to one and only one vector space $V_n(F)$.

c) Show that if a vector is orthogonal to $\xi_1, \xi_2, \xi_3 \ldots\ldots \xi_n$ then it is orthogonal to every vector in the subspace spanned by $\xi_1, \xi_2, \xi_3 \ldots\ldots \xi_n$.

Q5) a) Find the polynomial of degree two which is orthogonal to 1 and x on the euclidean vector space defined by $(f, g) = \int_0^1 f(x) g(x) dx$.

b) Find the orthonormal basis for the subspace of the Euclidean space (1, 0, −1), (0, 3, 4) and (1, 0, 1)

c) Find the basis for the orthogonal complement of the subspace spanned by (2, −1, 2) in the Euclidean three space.

SECTION - C

Q6) a) Find a linear transformation $T: V_2(R) \to V_3(R)$ such that T(−1, 1) = (−1, 0, 2) and T(2, 1) = (1, 2, 1).

b) Show that $T: R^3 \to R^2$ defined by $T(x, y, z) = (x + z, x + y + z)$ is a linear transformation.

c) Find the matrix of the linear transformation $T : V_2(R) \to V_3(R)$ defined by $T(x, y) = (x + y, x, 3x - y)$ relative to the basis $B_1 = \{(1, 1), (3, 1)\}$, $B_2 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$

Q7) a) Find a linear transformation of the matrix $\begin{bmatrix} -1 & 0 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ relative to the bases

$B_1 = \{(1, 0), (2, -1)\}, B_2 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$.

b) Find a linear transformation whose range is spanned by (1, 0, −1) and (1, 2, 2)

c) Find the eigen values and eigen vectors of the transformation $T(x, y) = (2x + 5y, 4x + 3y)$

Q8) a) Show that the correspondence $T(x, y, z) = (-y, -x, z)$ is an automorphism of $V_3(R)$. Find its order.

b) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$ using linear transformation.

c) Show that the product of any two linear transformation is alos a linear transformation.

Q9) a) Find the range, kernal, rank and nullity of the linear transformation $T(x, y, z) = (x, 2y, 3z)$

b) Show that the linear transformation $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (x - y, x - 2y)$ is non - singular.

c) Prove that the relation 'A' is similar to 'B' where A and B are square matrices is an equivalence relation.

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